Carbon Nanotubes dipole antenna Modeling: Comparison of electromagnetic Approach and Transmission Line Model

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Biography

Mourad AIDI is an Assistant Professor at Higher Institute of Computer Science and Multimedia of Gabes (ISIMG). He received the Master degree in Applied Physics from Higher School of Sciences and Techniques of Tunis and the MSc degree in energetic physics from National Institute of Applied Science and Technology (INSAT).

In 2016, he received the PhD degree in Telecommunications from the National Engineering School of Tunis. His research interest is in the electromagnetic modeling of nano-devices based on carbon nanotubes and graphene layers for telecommunication applications, Numerical methods, nano-antennas and RFID systems.

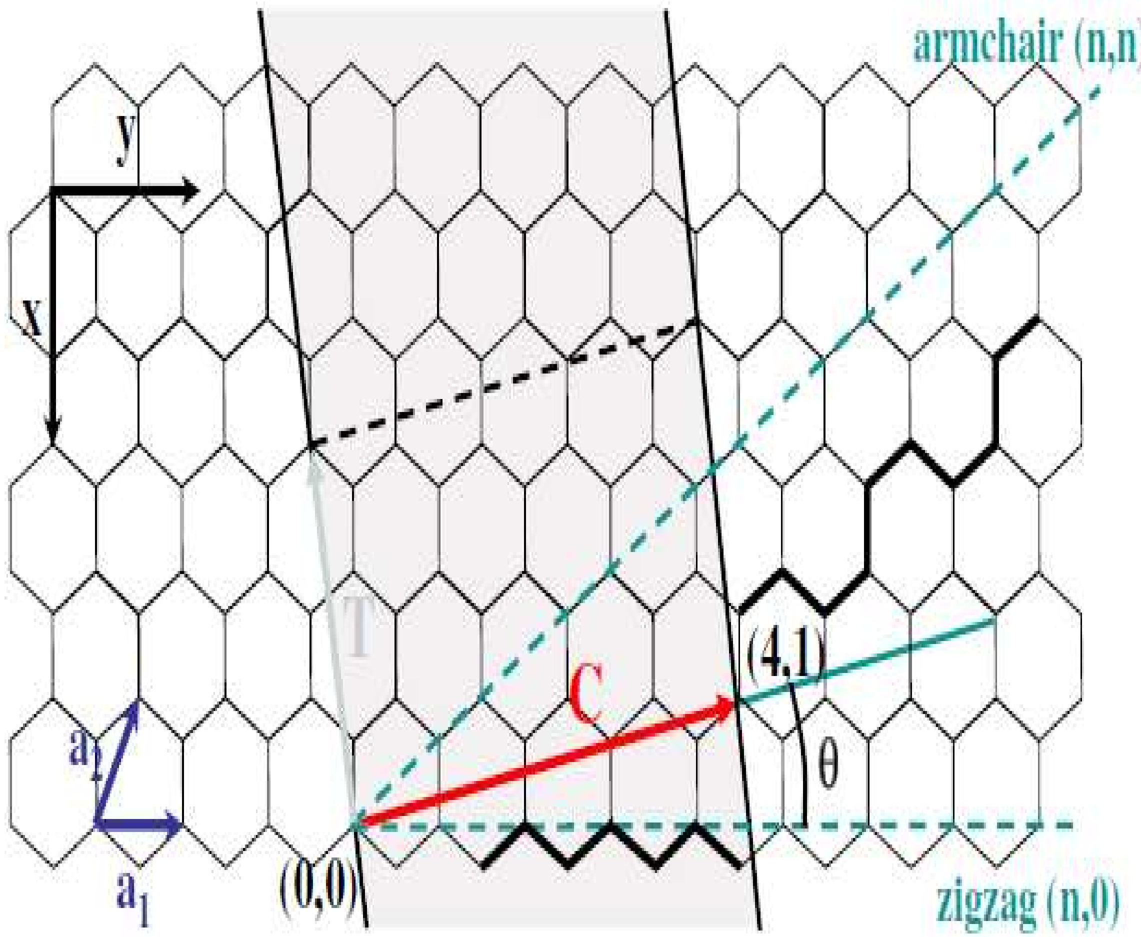
Abstract

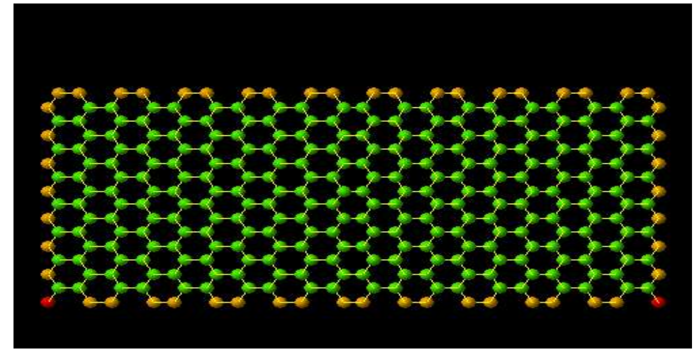
Fundamental properties of carbon nanotube antenna are firstly investigated to predict the antenna bundle response. The carbon nanotube effects are mathematically introduced via a quantum mechanical conductivity. This paper presents a new formulation based on integral equations system to study the coupled carbon nanotube antennas. The proposed integral equations system is numerically solved by the moments method. Each dipole antenna is excited at its center by a gap voltage source. The aim of the developed method is to investigate the antennas interaction effects for any coupling distance. The obtained input impedances, the current distributions and the antenna radiation patterns are in agreement with those obtained by the effective conductivity method or by the array factor method, according to the coupling distances.

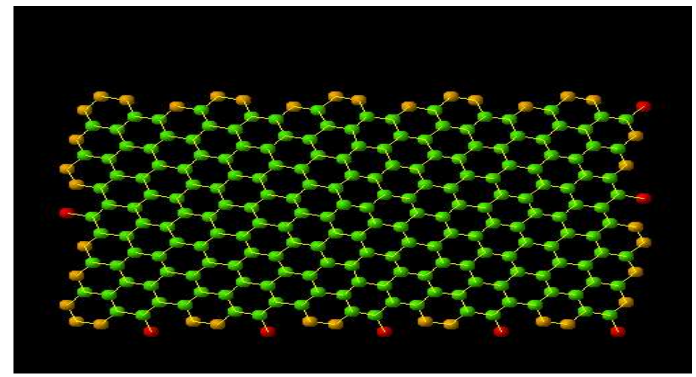
Keywords: Carbon nanotubes (CNT), dipole antennas, Integral equations, moments method .

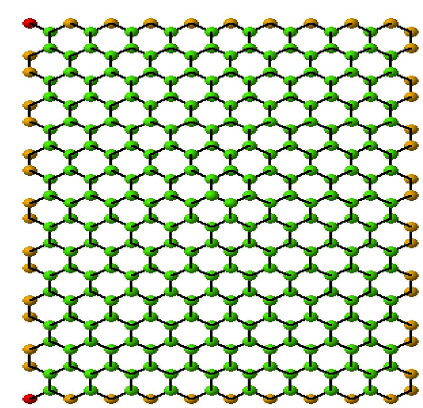
Crystallographic Structure of CNT

$$\vec{C} = n\vec{a_1} + m\vec{a_2}$$





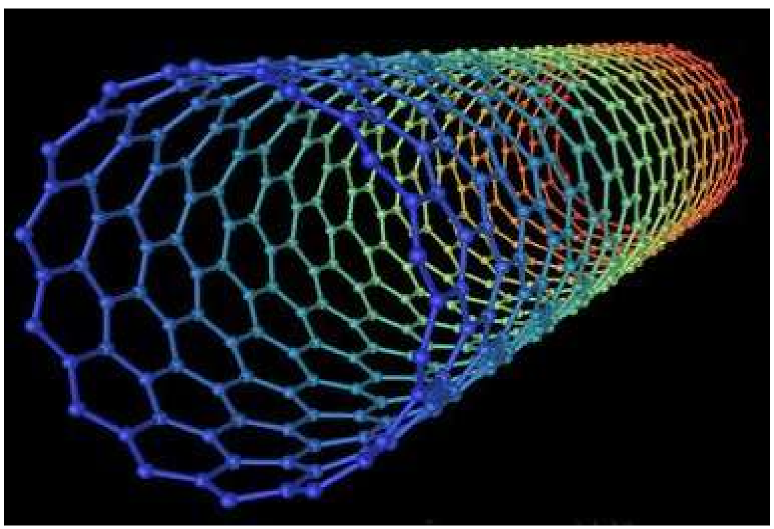




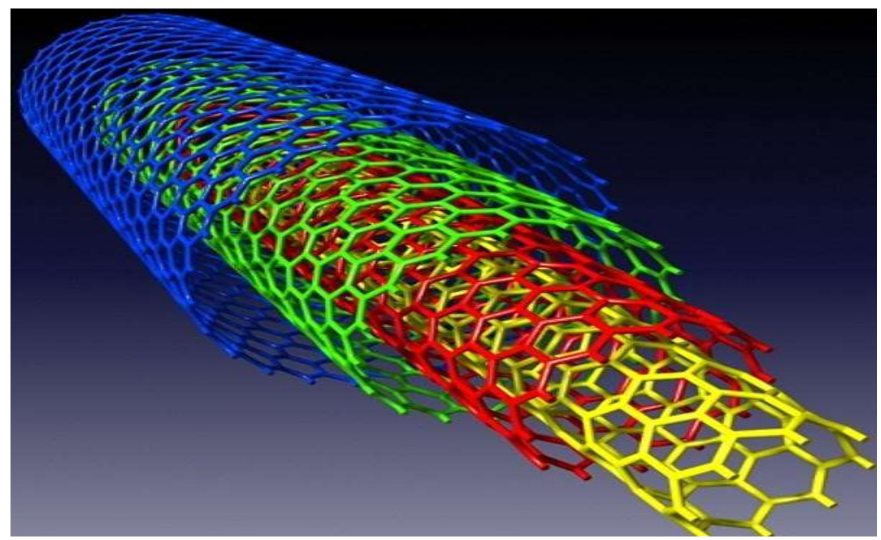
Kinds of Carbon Nanotubes

Carbon Nanotubes

Single-walled Carbon Nanotubes (d = 1-2 nm).



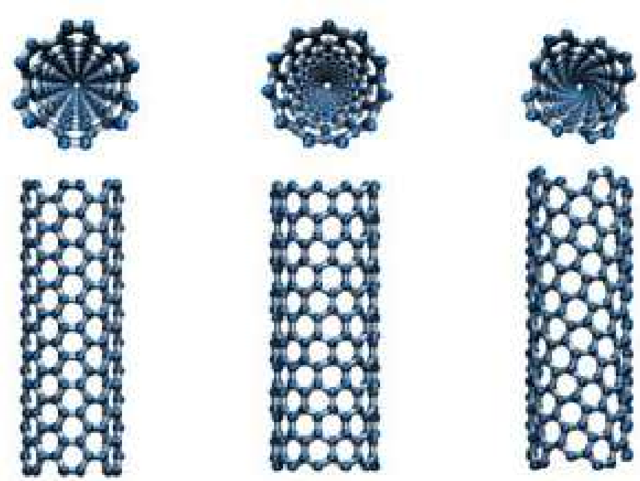
Multi-walled Carbon Nanotubes (d = 5-80 nm)



Electronic Properties of CNT

Carbon Nanotubes

2n+m=3q → **Metallics CNT**



2n+m≠3q → **Semiconductors CNT**

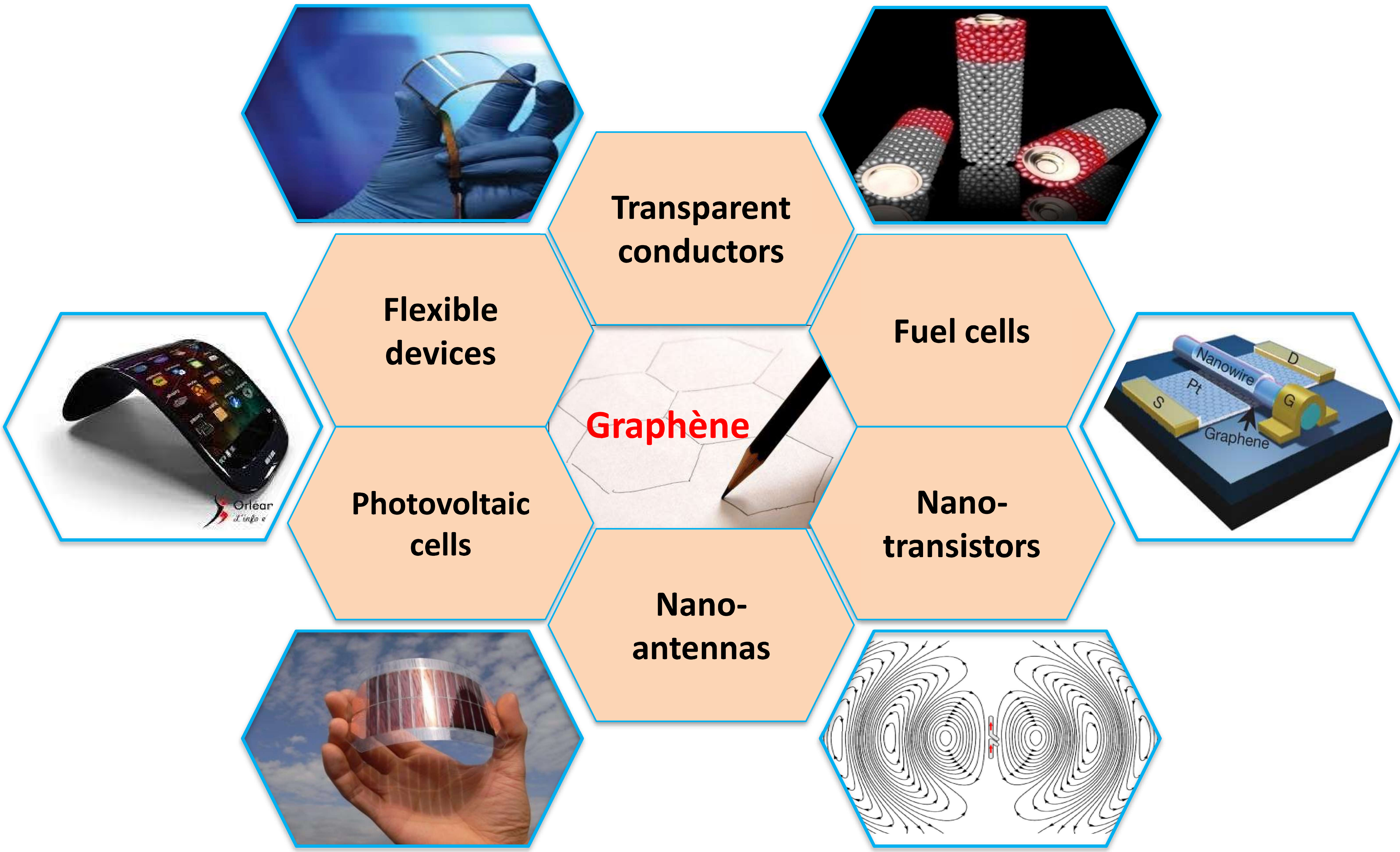
Electronic Properties of CNT

Material	CNT	Cu	Si
Resistivity (Ω.m)	~10^{-8}	$1.7. 10^{-8}$	$10·10^{-8}$
Max current density (A/cm^2)	~10^8	~10^6	impacted by thermal conductivity of substrate
Electron mobility ($cm^2V^{-1}s^{-1}$)	200000	32	1300

➡ The moving speed of the electrons in the CNT is 150 times greater than in silicon

➡ CNT is an emitters of fields (wave) to the nanometer scale

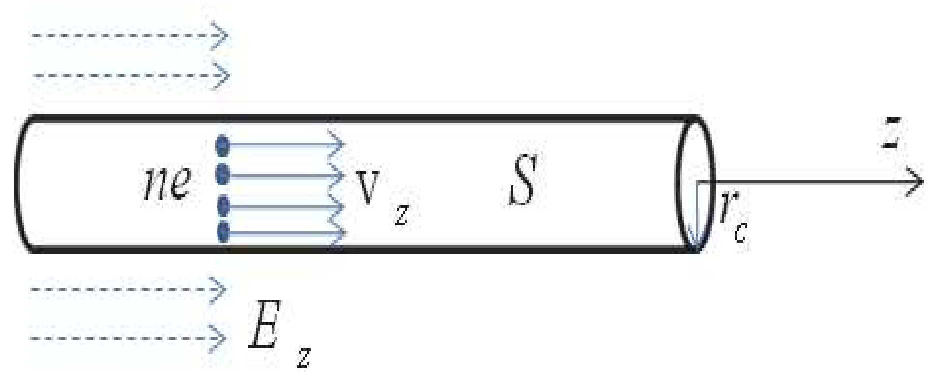
CNT Applications



Electron fluid model

Electron fluid model is presented to describe the linear response of SWNT to an applied electromagnetic field.

The motion of the perturbed π-electrons is modeled as a compressible charged fluid with friction.



The displacement of electron fluid is obeyed to the law of momentum conservation:

$$n_0 m_{eff} \frac{\partial V_Z}{\partial t} + \nu n_0 m_{eff} V_Z + \frac{\partial \delta p}{\partial z} = n_0 e E_Z$$

$$\delta p = m_{eff} v_F^2 \delta n$$

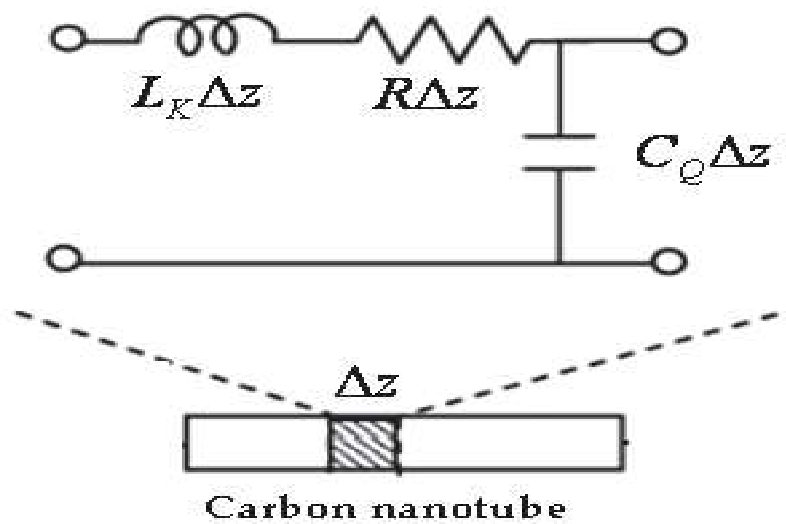
Transmission Lines Method for CNT Antenna Modeling

The longitudinal current: $I_Z = 2\pi . r . e . n_0 . V_Z$

The surface charge density: $q = 2\pi . r . e . n$

$$\frac{h}{8 v_F e^2} \frac{\partial I_Z}{\partial t} + \frac{v h}{8 v_F e^2} I_Z + \frac{v_F h}{8 e^2} \frac{\partial q}{\partial z} = E_Z$$

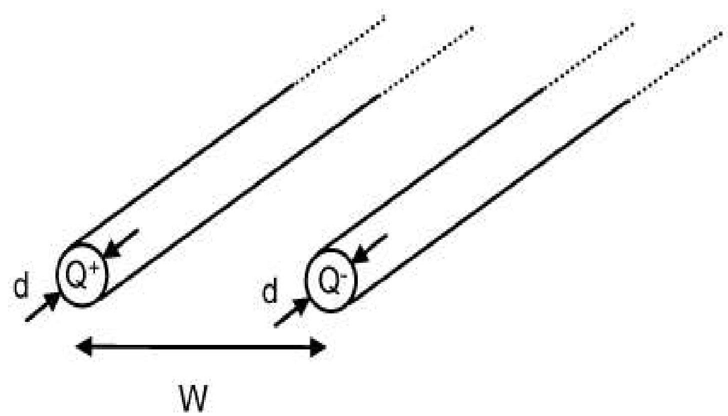
$$L_K = \frac{h}{8 v_F e^2} \qquad R = \frac{v h}{8 v_F e^2} \qquad C_Q = \frac{8 e^2}{v_F h}$$

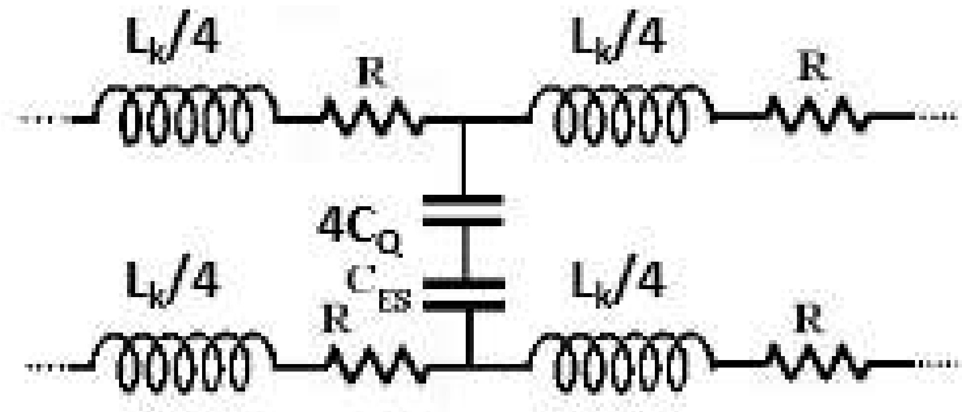


Transmission Lines Method for CNT Antenna Modeling

Equivalent circuit model for two CNTs

Equivalent circuit model would be the combination of the equivalent circuit of electron flow along CNT and the conventional circuit model based on electristatic capicitance and magnetic inductance .





$$\frac{L_M}{L_k} \sim 10^{-4} \qquad \frac{C_{ES}}{C_Q} \sim 1$$

$$\begin{cases} \dfrac{\partial^2 V(x)}{\partial x^2} - \gamma_p^2 . \mathrm{V(x)} = 0 \\ \dfrac{\partial^2 I(x)}{\partial x^2} - \gamma_p^2 . I(\mathrm{x}) = 0 \end{cases}$$

$$\gamma_p^2 = 2(R_Q + jw\frac{L_k}{4})(jC_T w)$$

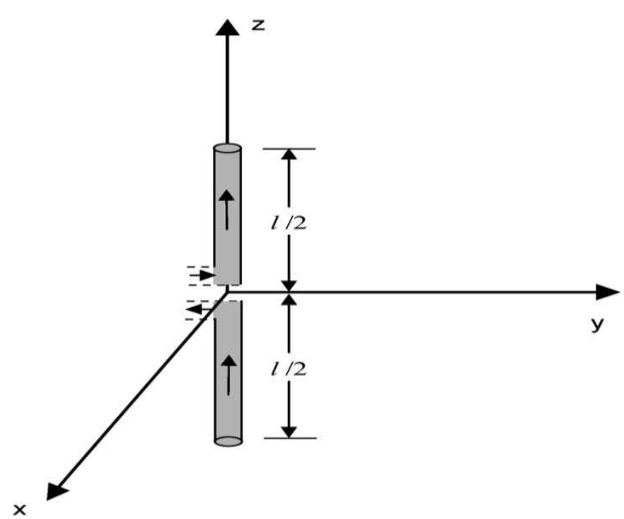
Current distribution

By appliying Khirchof's lows we find

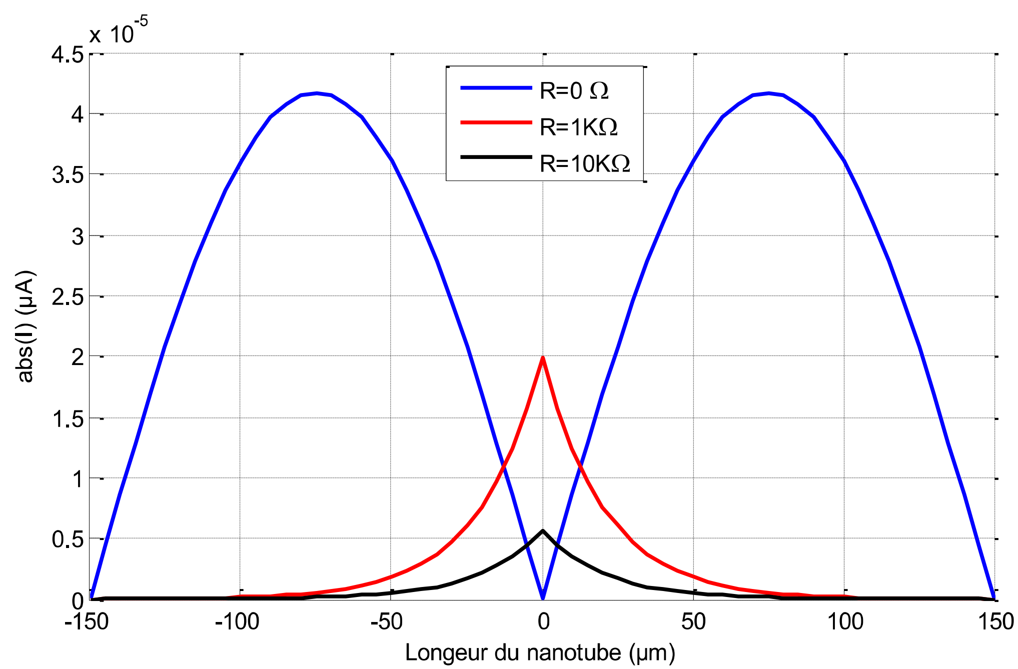
$$I(z)=\begin{cases}\frac{V_0^+}{Zc}\sinh\left[\gamma_p(\frac{L}{2}-z)\right]; if\ 0<z<\frac{L}{2}\\ \frac{V_0^+}{Zc}\sinh\left[\gamma_p(\frac{L}{2}+z)\right]; if\ -\frac{L}{2}<z<0\end{cases}$$

In the case of no loss (R=0):

$$I(z)=\begin{cases}\frac{V_0^+}{Zc}\sin\left[k_p(\frac{L}{2}-z)\right]; if\ 0<z<\frac{L}{2}\\ \frac{V_0^+}{Zc}\sin\left[k_p(\frac{L}{2}+z)\right]; if\ -\frac{L}{2}<z<0\end{cases}$$

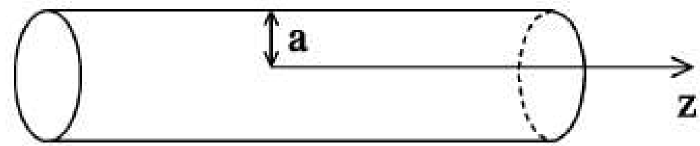


L=300µm et f=10GHz



Electromagnetic model for CNT dipole antenna

Dynamic Conductivity of CNT

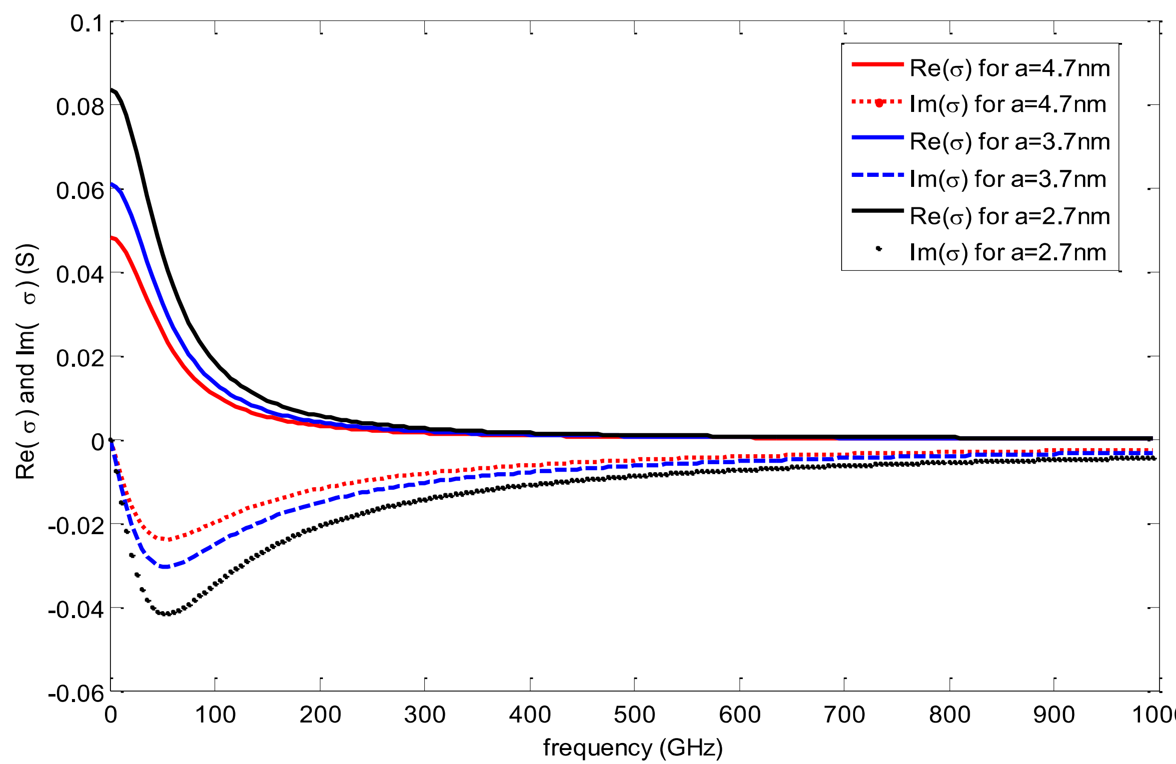


Dynamic conductivity of CNT represents a macroscopic quantity relating to the perturbation of electron flow along the CNT.

$$\sigma_{cn}(w) = \sigma_{zz}(w) \simeq -j\frac{2e^2 v_F}{\pi^2 \hbar a (w - j\nu)}$$

$$\upsilon = 3.10^{-12} s^{-1}$$
$$v_F = 9.71.10^5 ms^{-1}$$

$$Z_s = \frac{1}{2\pi a \sigma_{cn}(w)} = \frac{\pi \hbar \nu}{4e^2 v_F} + j\frac{\pi \hbar}{4e^2 v_F} w$$



Electromagnetic model for CNT dipole antenna

Integral Equation

Electric field continuity condition at the surface of CNT antenna:

$$E^{in} + E^{d} - Z_s J = 0$$

a is the antenna radius which will be in the order of nanometer, then we can approximate the current density as:

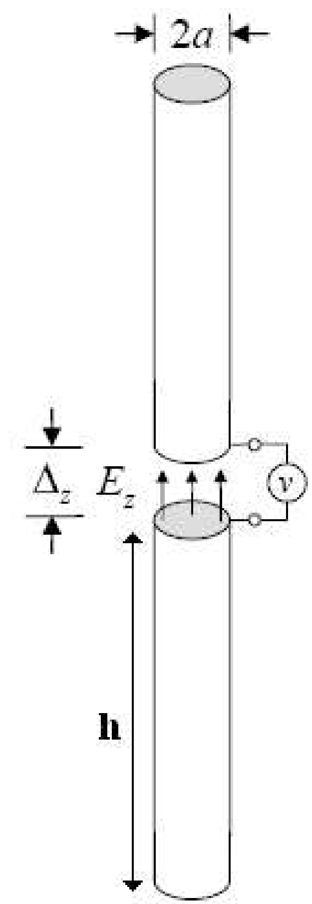
$$J(z') = \frac{I(z')}{2\pi a}$$

Radiated electric field can be expressed as:

$$E_z^d = \frac{1}{j4\pi w\varepsilon}\left(k^2 + \frac{\partial^2}{\partial z^2}\right)\int_{-h}^{h} \frac{e^{-jk\sqrt{(z-z')^2+a^2}}}{\sqrt{(z-z')^2+a^2}} I(z')dz'$$

Pocklington integral equation

$$\frac{1}{j4\pi w\varepsilon}\left(k^2 + \frac{\partial^2}{\partial z^2}\right)\int_{-h}^{h} G(z-z')I(z')dz' - \frac{Z_s}{2\pi a} I = -E^{in}$$



Electromagnetic model for CNT dipole antenna

The current along the CNT is expressed as the sum of the samples current I_n using the basis function f(z):

$$I(z) = \sum_{n=1}^{N} I_n f(z - z_n)$$

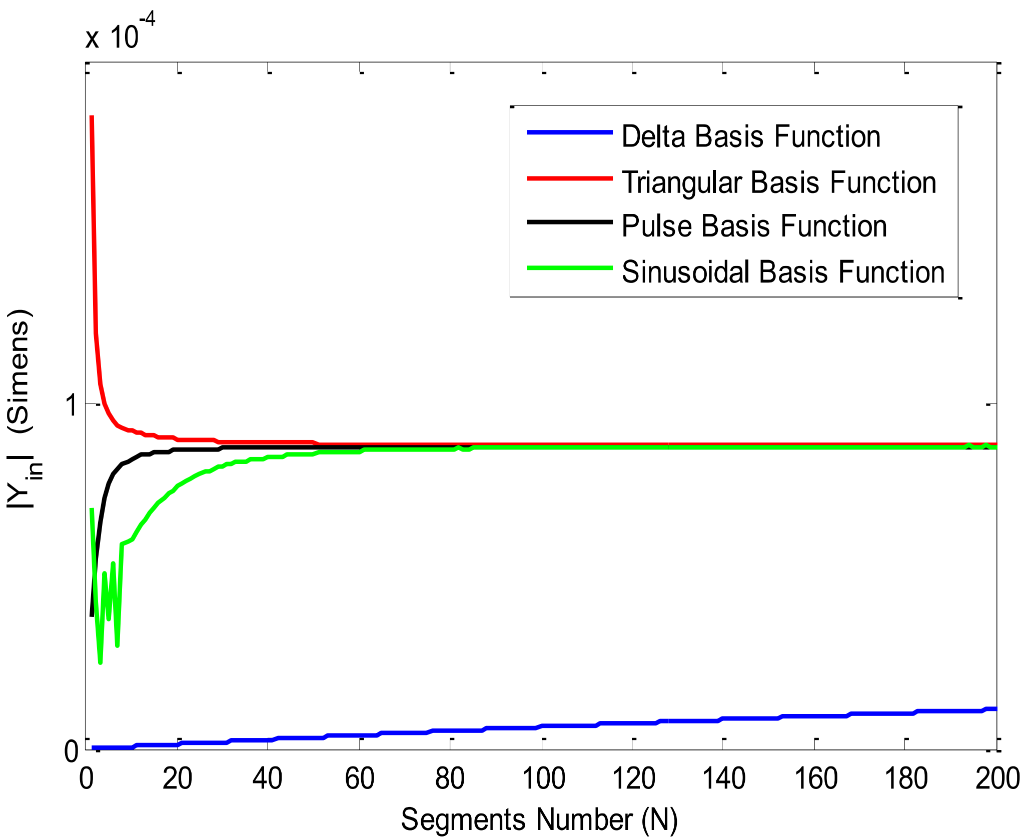
$$\Rightarrow \frac{1}{jw\varepsilon} \sum_{n=1}^{N} I_n \left\langle g_m, (k^2 + \frac{\partial^2}{\partial z^2}) \int_{-l/2}^{l/2} G(z,z') f(z - z_n) dz' \right\rangle - \sum_{n=1}^{N} \frac{Z_s}{2\pi a} I_n \langle g_m, f(z - z_n) \rangle = -\langle g_m, E_z^{in}(z) \rangle$$

$$\begin{bmatrix} Z_{11} & Z_{12} & . & . & Z_{1N} \\ Z_{21} & Z_{22} & . & . & Z_{2N} \\ . & & . & & . \\ . & & & . & . \\ Z_{N1} & Z_{N2} & . & . & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ . \\ . \\ . \\ I_N \end{bmatrix} = - \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_N \end{bmatrix} \quad \Rightarrow \quad [I] = [Z]^{-1} [V]$$

$$Z_{mn} = \frac{1}{jw\varepsilon} \left\langle g_m, (k^2 + \frac{\partial^2}{\partial z^2}) \int_{-l/2}^{l/2} G(z,z') f(z - z_n) dz' \right\rangle - \frac{Z_s}{2\pi a} \langle g_m, f(z - z_n) \rangle$$

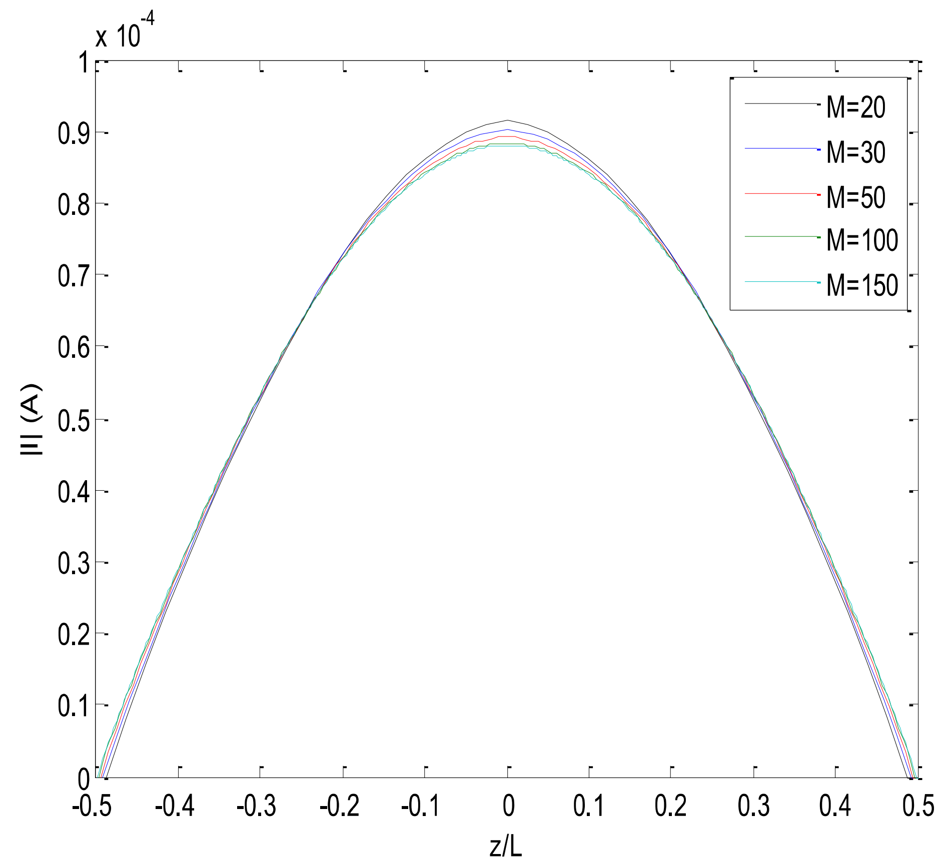
Numerical Results

Convergence study



Input admittance for diffrent bases function number (L=20µm et f=160GHz)

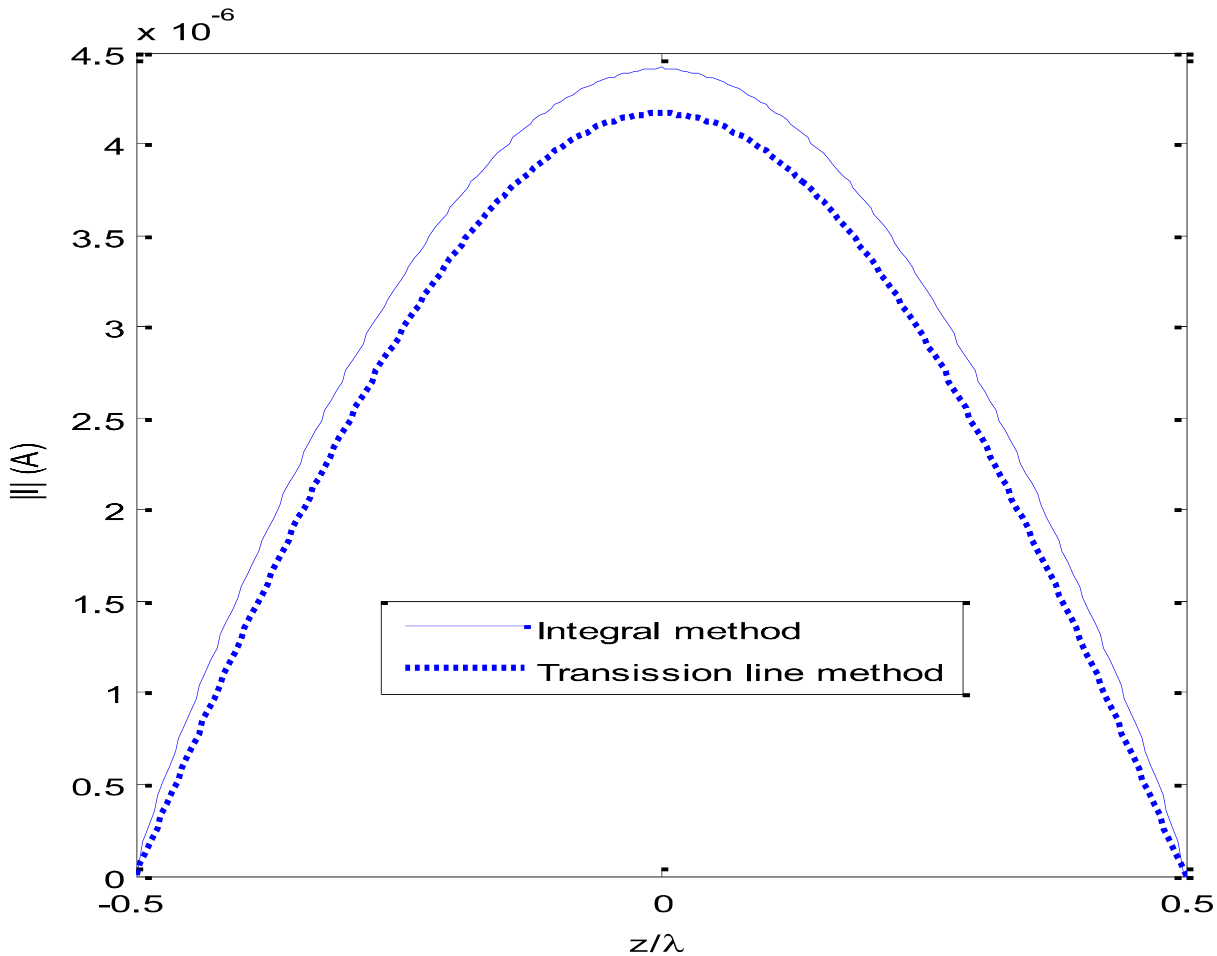
Triangular bases function



Current distribution for different bases function number (L=20µm et f=160GHz)

Numerical Results

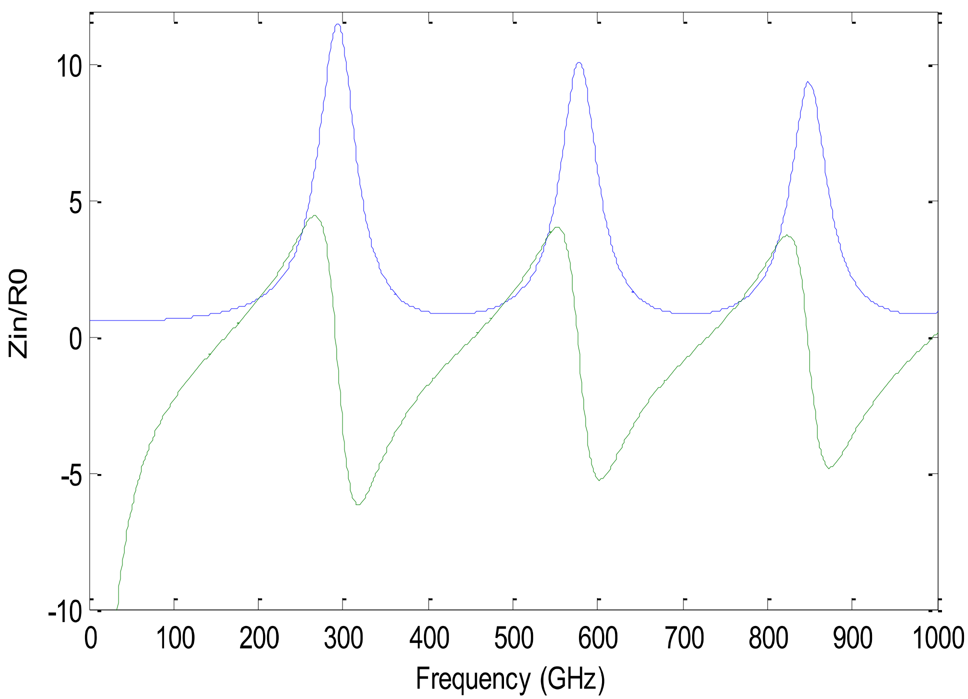
Current Distribution



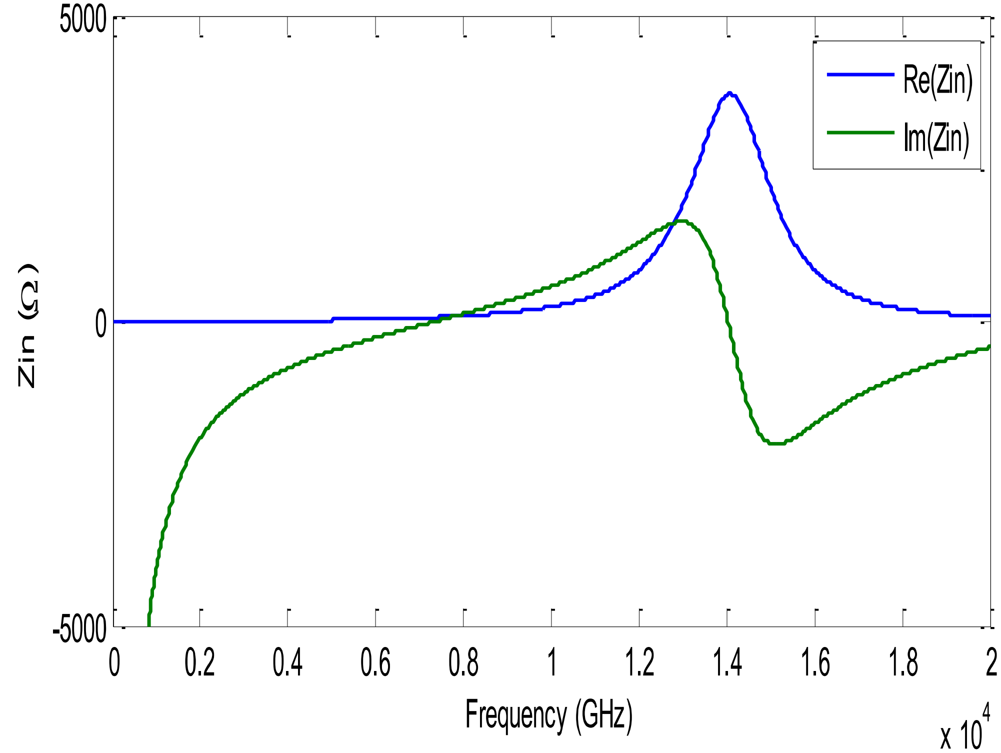
Current distribution for CNT antenna with radius (a=2.7nm) and length l=λp/2 for operating frequency f=10GHz

Numerical Results

Input Impedance



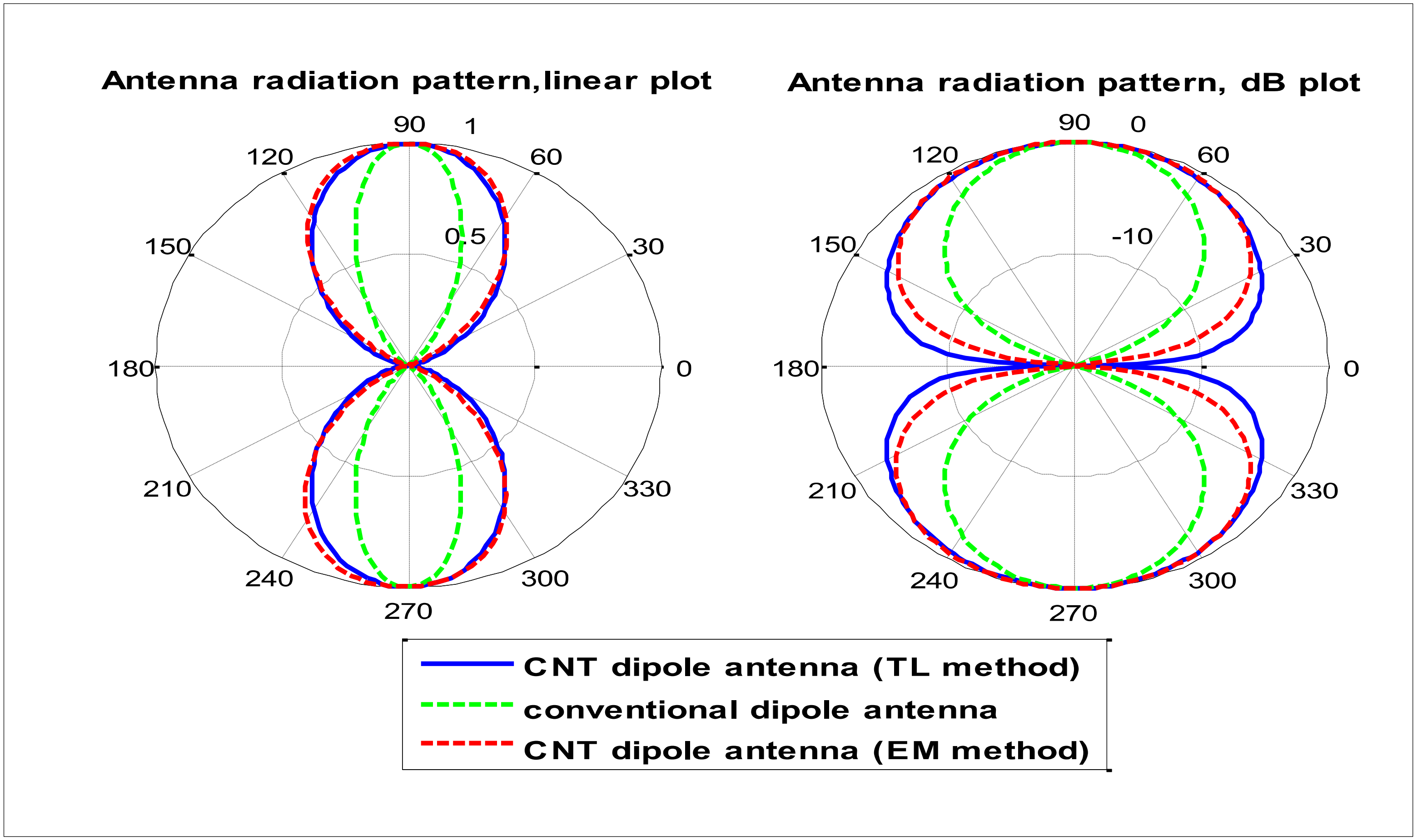
Input impedance for a CNT dipole antenna: L=20µm, a=2.71nm, R0=12.9KΩ



Input impedance for a perfect conductor dipole antenna: L=20µm, a=2.71nm,

Numerical Results

Antenna Radiation Pattern

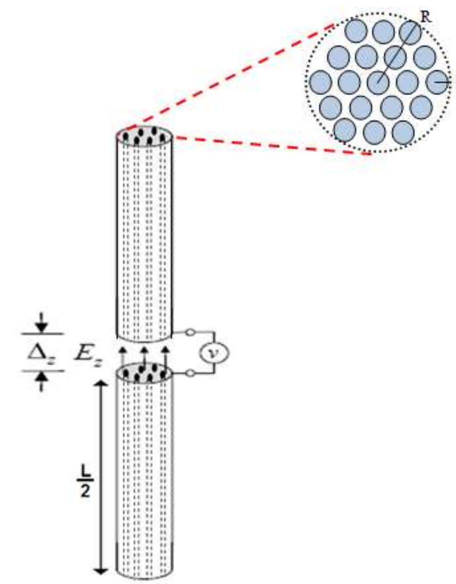


Coupled NTC Antennas

Strongly coupled antennas: Effective Conductivity

$$\sigma_{eff} \simeq \frac{N\sigma_{cn}(w)a}{R} \Rightarrow Z_s = \frac{1}{2\pi a \sigma_{eff}}$$

$$(k^2 + \frac{\partial^2}{\partial z^2})\int_{-L/2}^{L/2} \frac{e^{-jk\sqrt{(z-z')^2+a^2}}}{\sqrt{(z-z')^2+a^2}} I(z')dz' = j4\pi w\varepsilon(Z_s I(z) - E_z^{in}(z))$$



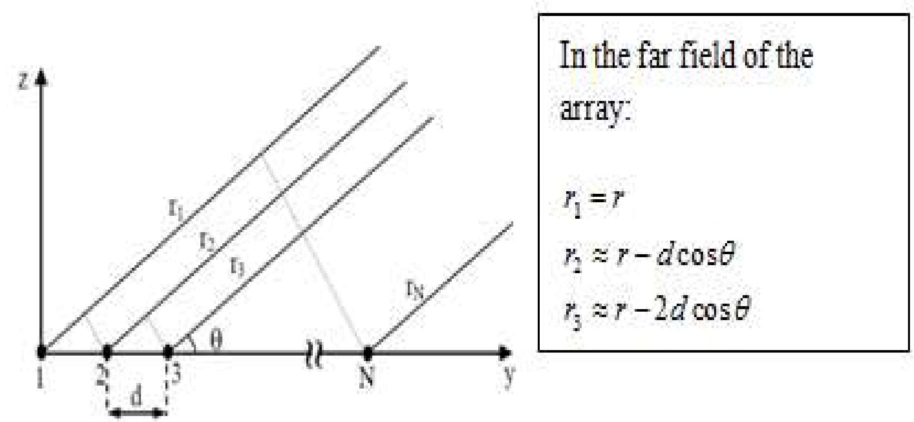
Isolated antennas: Array Factor (A.F)

$$A.F = \sum_{n=1}^{N} e^{j(n-1)\psi}$$

$$\psi = kd\cos(\theta) + \delta$$

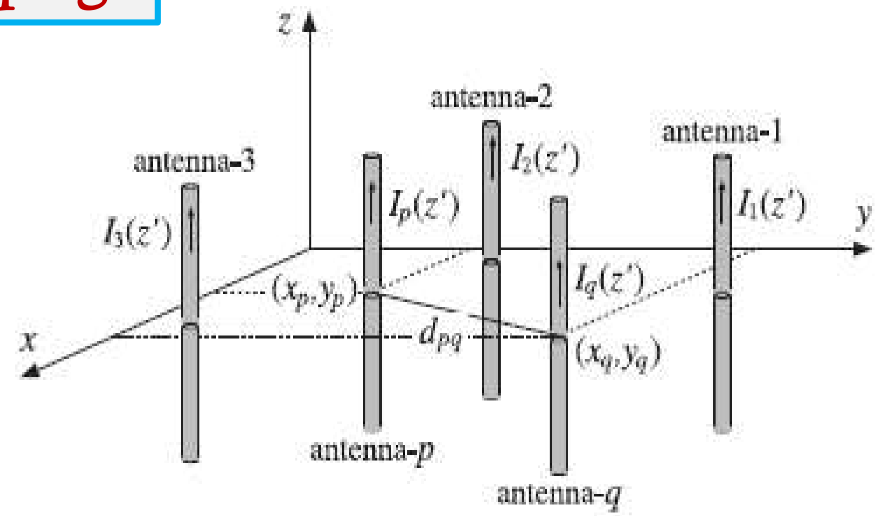
Uniform linear array antennas of N elements:

$$A.F = \frac{\sin(N\frac{\psi}{2})}{\sin(\frac{\psi}{2})} e^{j(N-1)\frac{\psi}{2}}$$



Coupled NTC Antennas

Formulation Intégrale pour l'Etude du couplage



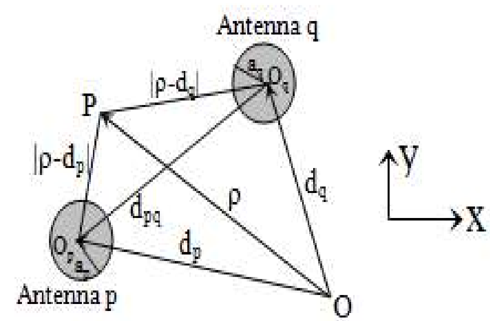
At the surface of each antenna, the electric field boudary condition is expressed as:

$$\begin{cases} E_{11}^{d} + E_{21}^{d} + E_{31}^{d} + \dots\dots + E_{N1}^{d} - Z_s I_1 = -E_1^{in} \\ E_{12}^{d} + E_{22}^{d} + E_{32}^{d} + \dots\dots + E_{N2}^{d} - Z_s I_2 = -E_2^{in} \\ . \\ . \\ . \\ E_{1N}^{d} + E_{2N}^{d} + E_{3N}^{d} + \dots\dots + E_{NN}^{d} - Z_s I_N = -E_N^{in} \end{cases}$$

$$E_{pq}^{d}(z) = \left(\frac{\partial^2}{\partial z^2} + k^2\right) \int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{e^{-jkR_{pq}}}{R_{pq}} I_q(z')dz' = G_{pq} I_q$$

Where: $G_{pq} = \frac{1}{j4\pi w\varepsilon}\left(k^2 + \frac{\partial^2}{\partial z^2}\right)\int_{-h}^{h} \frac{e^{-jkR_{pq}}}{R_{pq}} dz'$

$$R_{pq} = \sqrt{(z - z')^2 + d_{pq}^{\;2}}$$



$$\begin{cases} d_{pq} = \sqrt{(x_p - x_q)^2 + (y_p - y_q)^2} \\ d_{pq} = a_p \end{cases}$$

Coupled NTC Antennas

$$\Rightarrow \begin{cases} \left\langle g_{1n},(\hat{G}_{11}-Z_s)I_1\right\rangle+\left\langle g_{1n},\hat{G}_{21}I_2\right\rangle+\left\langle g_{1n},\hat{G}_{31}I_3\right\rangle+........+\left\langle g_{1n},\hat{G}_{N1}I_N\right\rangle=-\left\langle g_{1n},E_1^{in}\right\rangle \\ \left\langle g_{2n},\hat{G}_{21}I_1\right\rangle+\left\langle g_{2n},(\hat{G}_{22}-Z_s)I_2\right\rangle+\left\langle g_{2n},\hat{G}_{32}I_3\right\rangle+........+\left\langle g_{2n},\hat{G}_{N3}I_N\right\rangle=-\left\langle g_{2n},E_2^{in}\right\rangle \\ . \\ . \\ . \\ \left\langle g_{Nn},\hat{G}_{N1}I_1\right\rangle+\left\langle g_{Nn},\hat{G}_{N2}I_1\right\rangle+\left\langle g_{Nn},\hat{G}_{N3}I_1\right\rangle+........+\left\langle g_{Nn},(\hat{G}_{NN}-Z_s)I_N\right\rangle=-\left\langle g_{Nn},E_N^{in}\right\rangle \end{cases}$$

The current can be expressed as a series of basis functionsf(z-z'):

$$I_q(z)=\sum_{m=-N}^{N} I_{qm} f(z-z_m)$$

$$\Rightarrow \underbrace{\begin{bmatrix} \left[\left\langle g_{1n},(\hat{G}_{11}-Z_s)f_{1m}\right\rangle\right] & . & . & . & \left[\left\langle g_{1n},\hat{G}_{N1}f_{1m}\right\rangle\right] \\ . & & . & & . \\ . & & & . & . \\ . & & & . & . \\ \left[\left\langle g_{Nn},\hat{G}_{1N}f_{Nm}\right\rangle\right] & . & . & . & \left[\left\langle g_{nN},(\hat{G}_{NN}-Z_s)f_{Nm}\right\rangle\right] \end{bmatrix}}_{Z} \underbrace{\begin{bmatrix} I_1 \\ . \\ . \\ . \\ I_N \end{bmatrix}}_{I} = - \underbrace{\begin{bmatrix} \left[\left\langle g_{1n},E_1^{in}\right\rangle\right] \\ . \\ . \\ . \\ \left[\left\langle g_{Nn},E_N^{in}\right\rangle\right] \end{bmatrix}}_{E} \Rightarrow I = Z^{-1}E$$

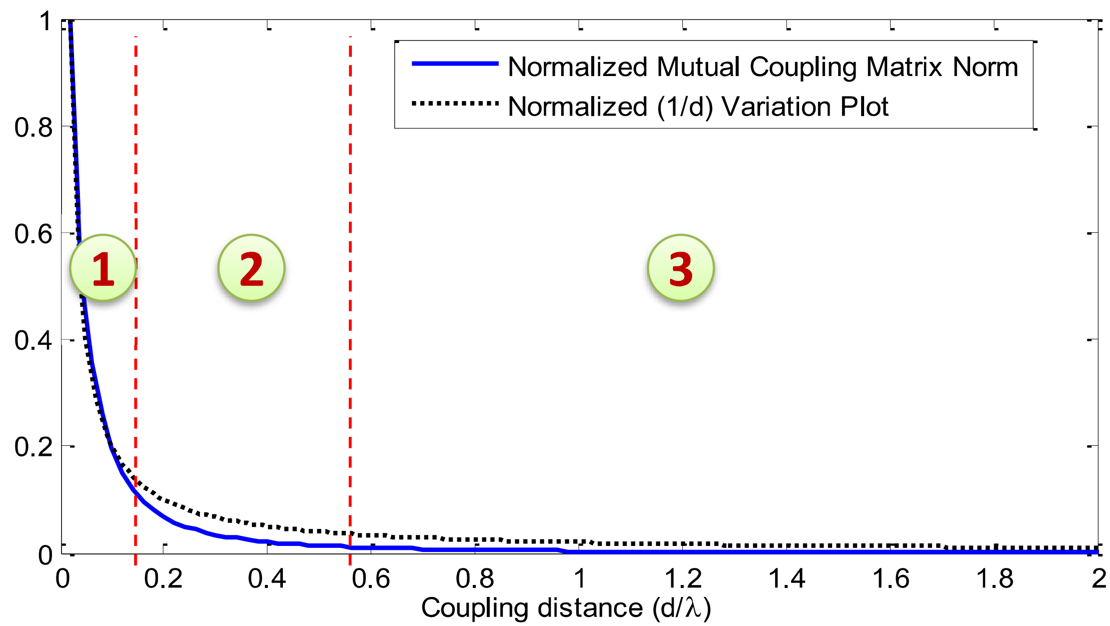
Coupled NTC Antennas

Coupling Distance Effect

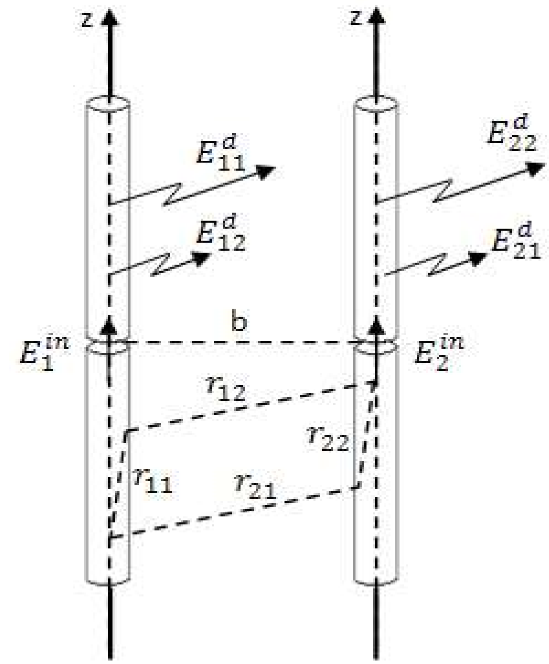
$$\begin{pmatrix} \left[\langle g_{n1},(G_{11}+Z_s)g_{m1}\rangle\right] & \left[\langle g_{n1},G_{21}g_{m2}\rangle\right] \\ \left[\langle g_{n2},G_{12}g_{m1}\rangle\right] & \left[\langle g_{n2},(G_{22}+Z_s)g_{m2}\rangle\right] \end{pmatrix} \begin{pmatrix} [I_{m1}] \\ [I_{m2}] \end{pmatrix} = - \begin{pmatrix} \left[\langle g_{n1},E_1^{in}\rangle\right] \\ \left[\langle g_{n2},E_2^{in}\rangle\right] \end{pmatrix}$$

Mutual coupling matrix

Self coupling matrix

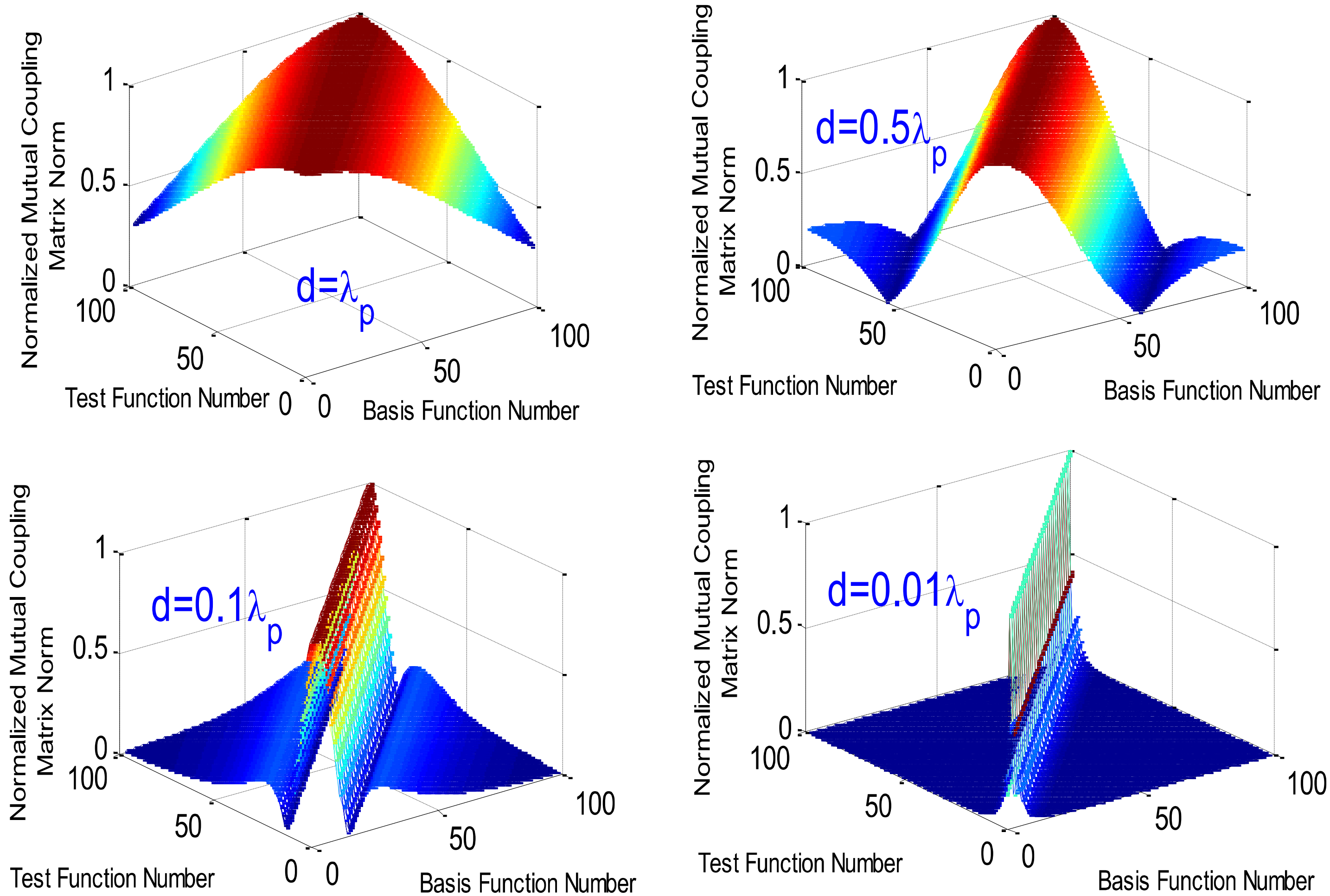


Mutual coupling matrix norm as a function of separating distance



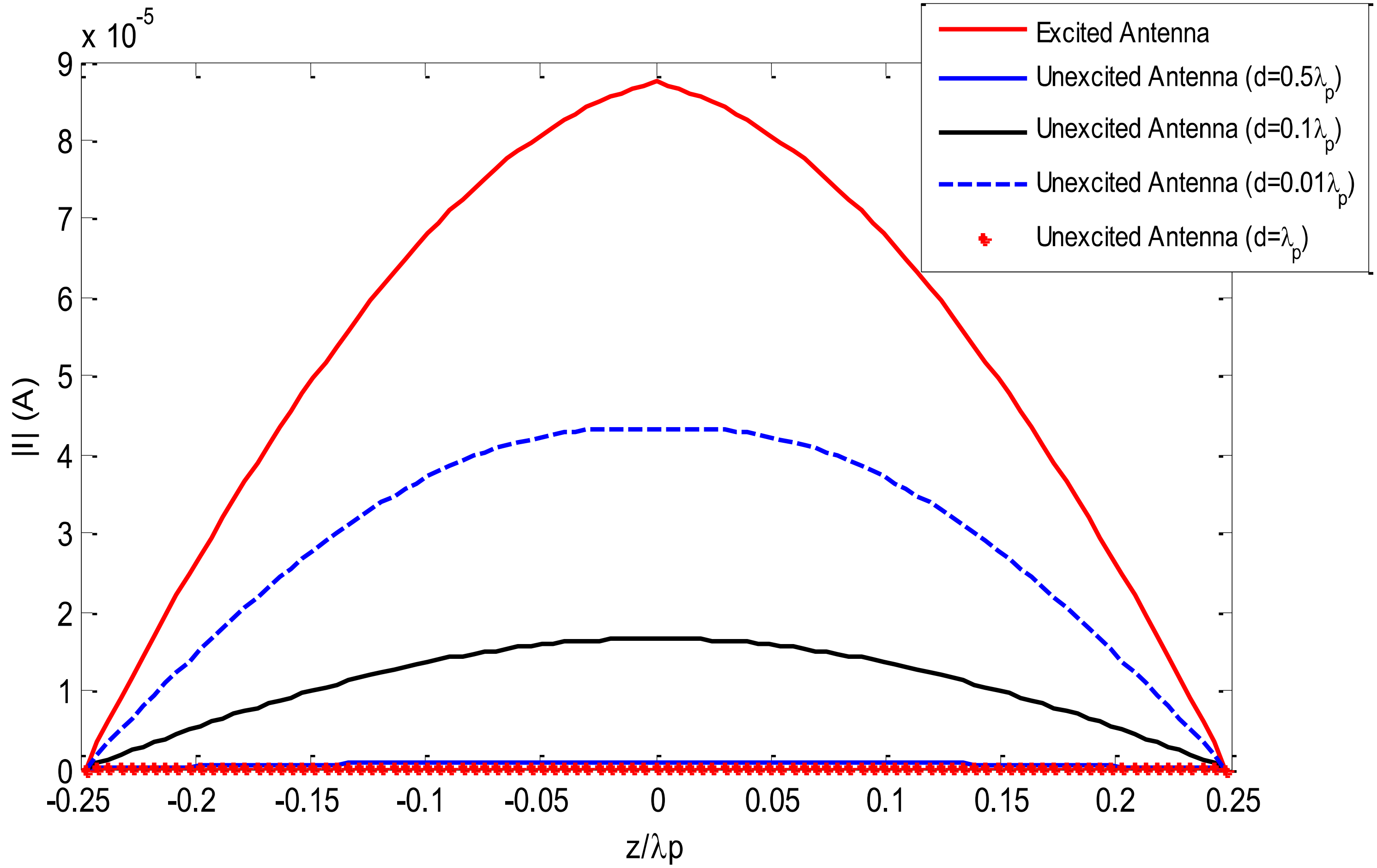
L=20µm
a=2.71nm
f=150GHz

Coupled NTC Antennas



Mutual coupling matrix norm for different separating distances

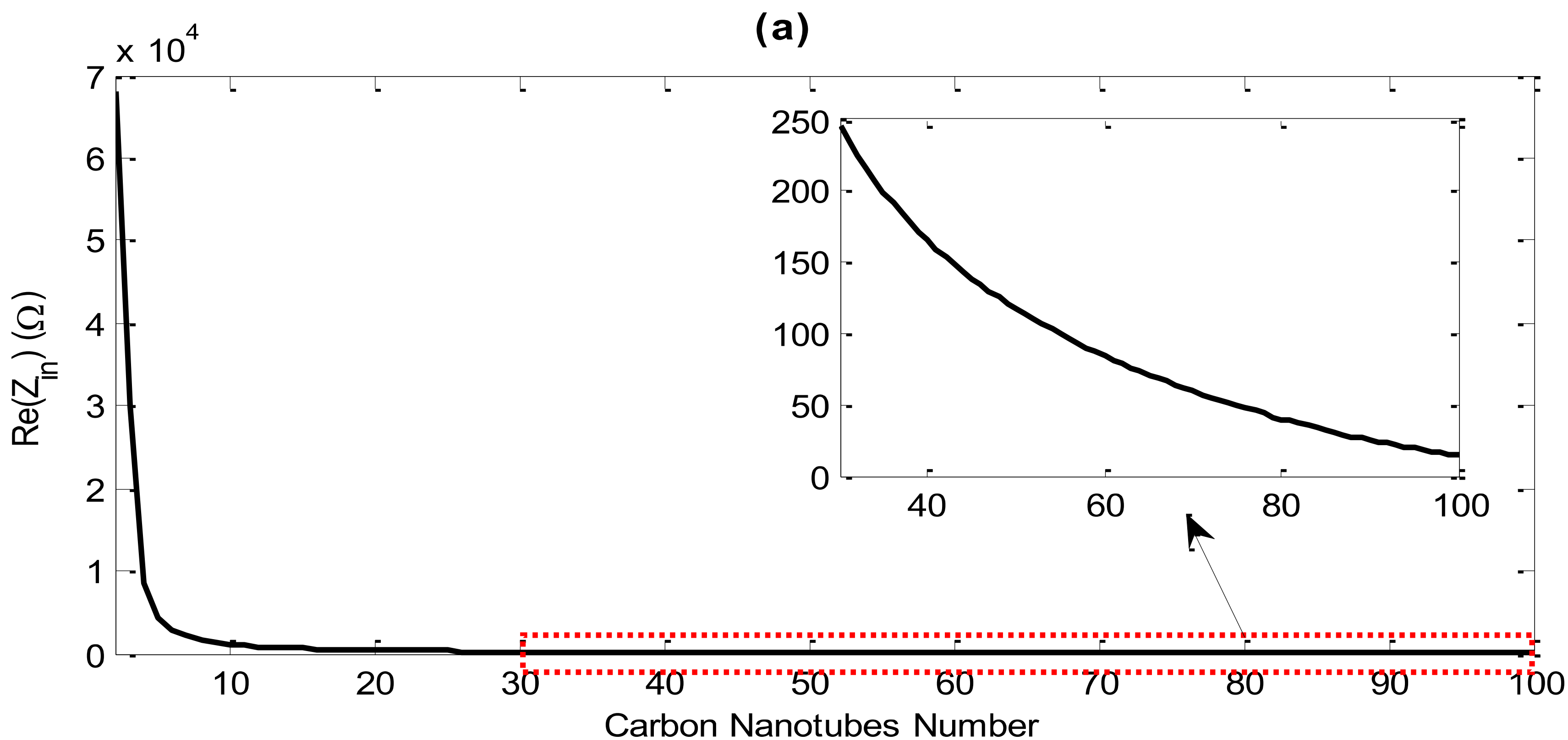
Coupled NTC Antennas



Current distribution for different separating distances

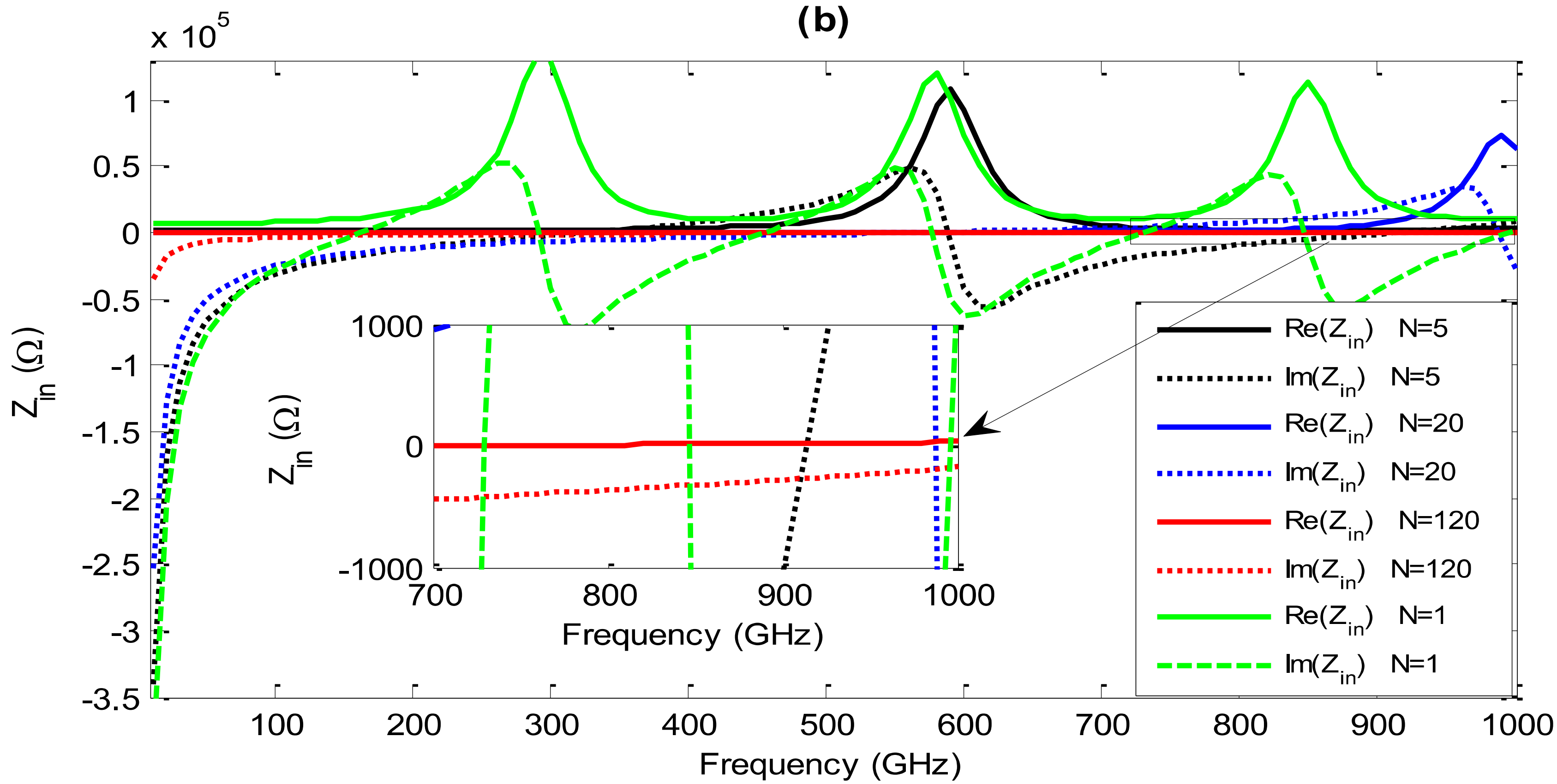
Coupled NTC Antennas

Antenna Number Effect



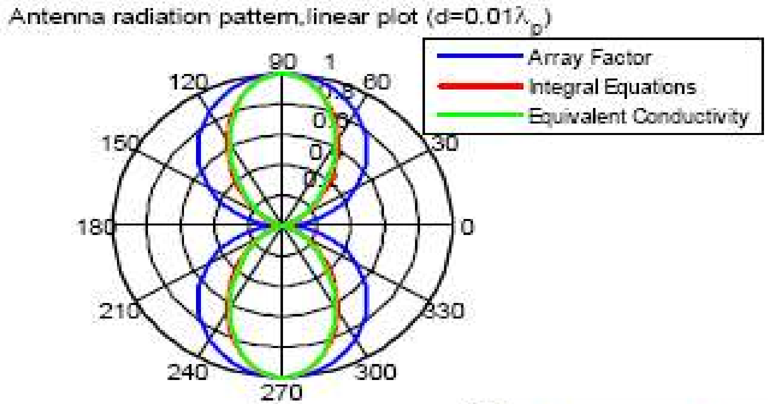
Input impedance as function of CNT antenna number

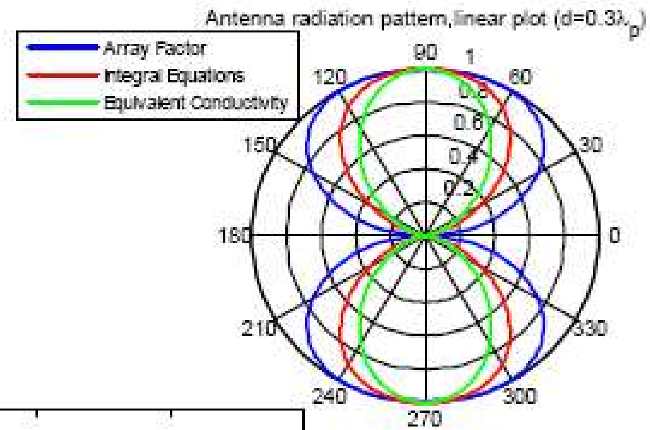
Coupled NTC Antennas

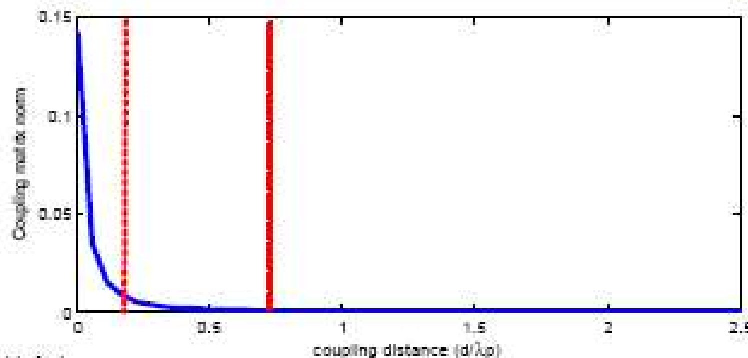


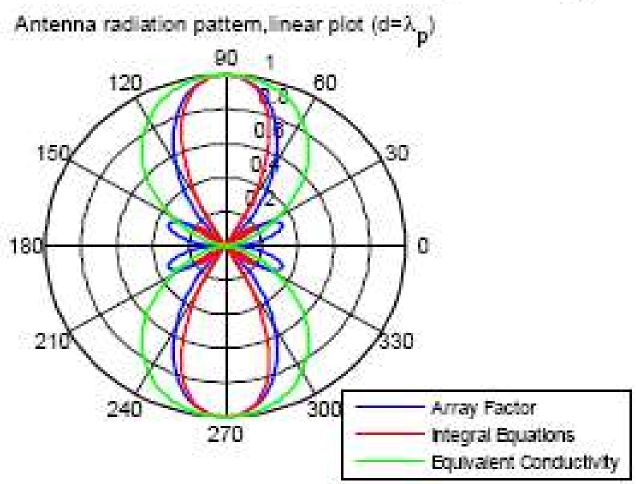
Input impedance as function of CNT antenna number

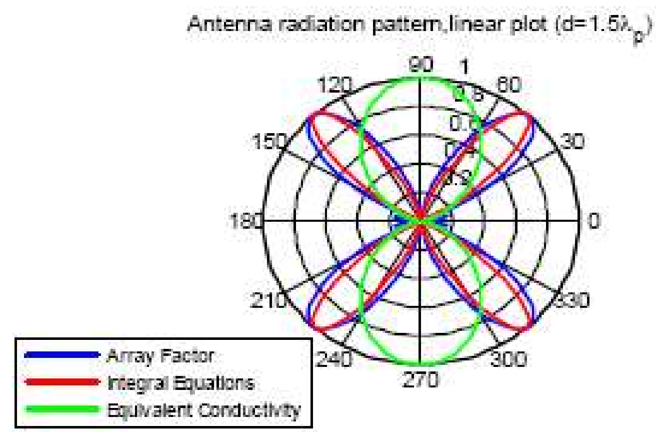
Coupled NTC Antennas











Coupled NTC Antennas

- **For highly coupled CNT antennas, the effective conductivity approach and the proposed integral method give the same antenna radiation patterns.**
- **For isolated CNT antennas, the array factor approach and the proposed integral method give the same antenna radiation patterns.**
- **For weakly coupled CNT antennas, the three methods behave differently.**

➡ **The three methods are in agreement according to the coupling distance.**

➡ **The proposed integral formulation can be also accurately applied for any coupling distance.**

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